1. 1. Yes because every input has a single output.
   2. No. Let . That means . Since and , then when and , . So there is more than one output for .
   3. No. Let . Since , then . So when then , so or . Thus there is more than one output for .
   4. Yes. Suppose and . That is, and . Since is transitive, then . Thus . Thus is a function.
   5. Yes. By the definition of a function, . Since the empty set has no or , the statement is vacuously true. Thus is a function.
2. 1. Let . Suppose . That is, . Subtracting from the RHS and adding it to the LHS gives you . Factoring out the on the LHS gives you . Dividing on both sides gives you . Thus . Therefore is one-to-one.

Let . that . Let with . That is, . So ⇒⇐. Hence . Therefore is not onto.

* 1. Let . Suppose . That is, . Subtracting from the LHS and adding it to the RHS gives us . Therefore is one-to-one.

Let and let . So . Thus . Then . So . Therefore is onto.

* 1. Let . Suppose . That is, . Subtracting from the LHS and adding it to the RHS gives us . Therefore is one-to-one.

Let . that . Let with . That is, . So Hence . Therefore is not onto.

* 1. Let u = 2 and let v = 3. So and . Thus but . So . Therefore is not one-to-one.

Let and let x = 2y. So . Thus . Then . So . Therefore is onto.

1. 1. 1. is a function:

Suppose that and . By definition of the relation, and. So . Subtracting x from the LHS and adding it to the RHS gives us . Therefore f is a function.

* + 1. :

Let . By the definition of the relation . So .

On the other hand, let . Then there is an integer such that , namely , or . So .

Therefore .

* + 1. F ⊆ ℤ:

Let . By definition of the relation, . So .

* 1. .
  2. Let . By the definition of the relation, and , or and . Suppose That is, Multiplying both sides by gives us . So is one-to-one.
  3. Let and let . So . So . So . Therefore is onto.